INDIAN STATISTICAL INSTITUTE
CHENNAI CENTRE
M. Stat. (NB Stream) – Semester I
2016 – 2018
Linear Algebra and Linear Models
Final Examination (Linear Algebra)

Total Marks: 50 **Duration:** 3 hours

1. Let $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$, where A, B, C and D are square matrices of the same order. Show that if A is nonsingular and C commutes with A then $|M| = |AD - CB|$. [5]
2. Define (i) inner product (ii) norm (iii) orthogonal projector [2+2+2]
3. Find all inner products and norms on $\mathbb{C}^1$. [2+2]
4. Let A and B be orthogonal projectors of order n. Show that $A+B$ is an orthogonal projector iff $\mathscr{C}(A) \perp \mathscr{C}(B)$ [5]
5. Prove or disprove the following.
   (a) Let A be a matrix of order n. If k is the largest integer such that λ^k divides $\chi_A(\lambda)$ then $\rho(A) = n - k$. [3]
   (b) If A is an idempotent matrix then A is semi-simple. [3]
6. Let A and B be matrices of order n. Show that if B is nonsingular then AB and BA are similar. [4]
7. Define singular value decomposition of a matrix. Show that every matrix has a singular value decomposition. [2+4]
8. Show that a real symmetric matrix A is p.d. iff all its leading principal minors are positive. [5]
9. Show that a real symmetric matrix A is n.n.d. iff it has a rank-factorization of the form (P, P^T) for some matrix P. [5]
10. Show that if A is an n.n.d. matrix then $A = B^2$ for some matrix B. [4]